Lecture 4

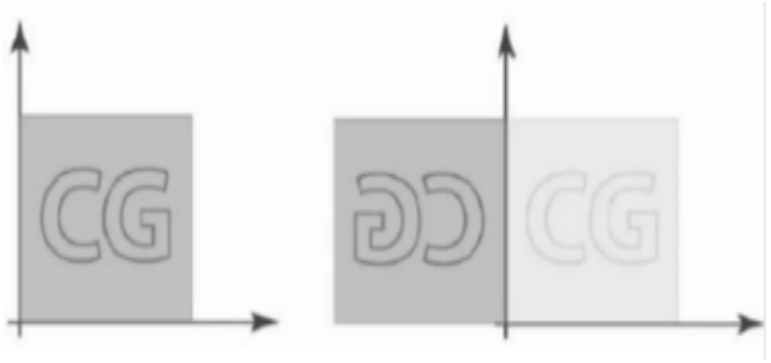
Reflection

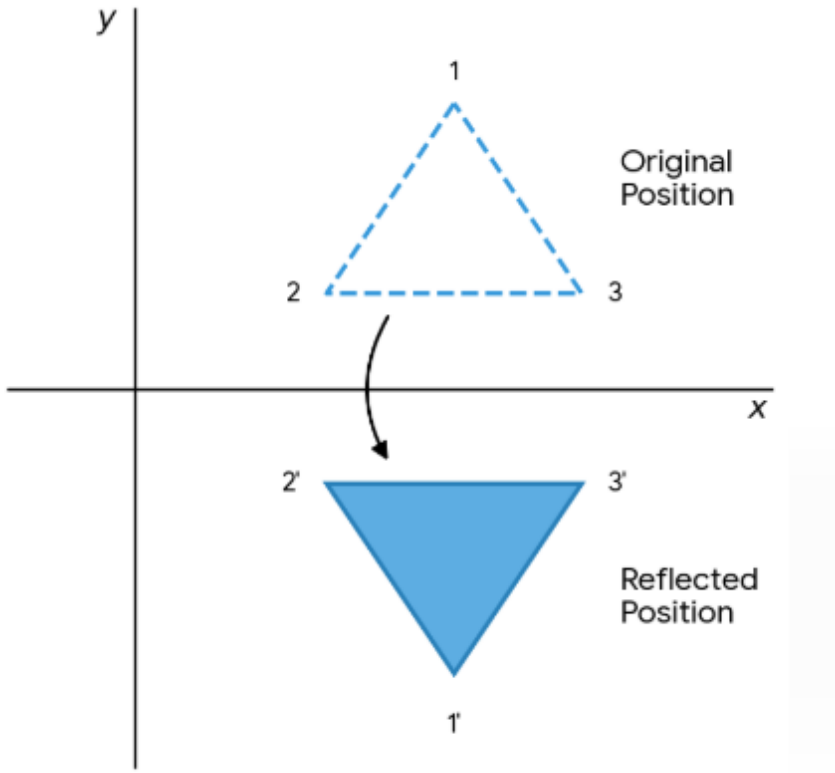
Reflection

- Transformation that produces a mirror image of an object.
- Image is generated relative to an axis of reflection by rotating the object 180 about the reflection x-axis.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

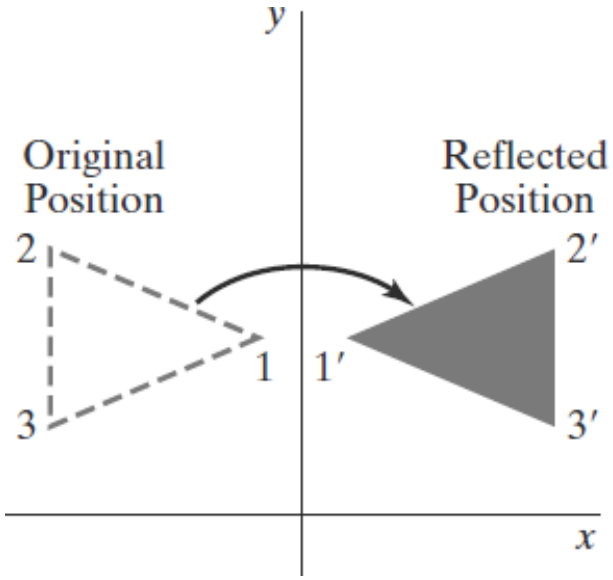
$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$





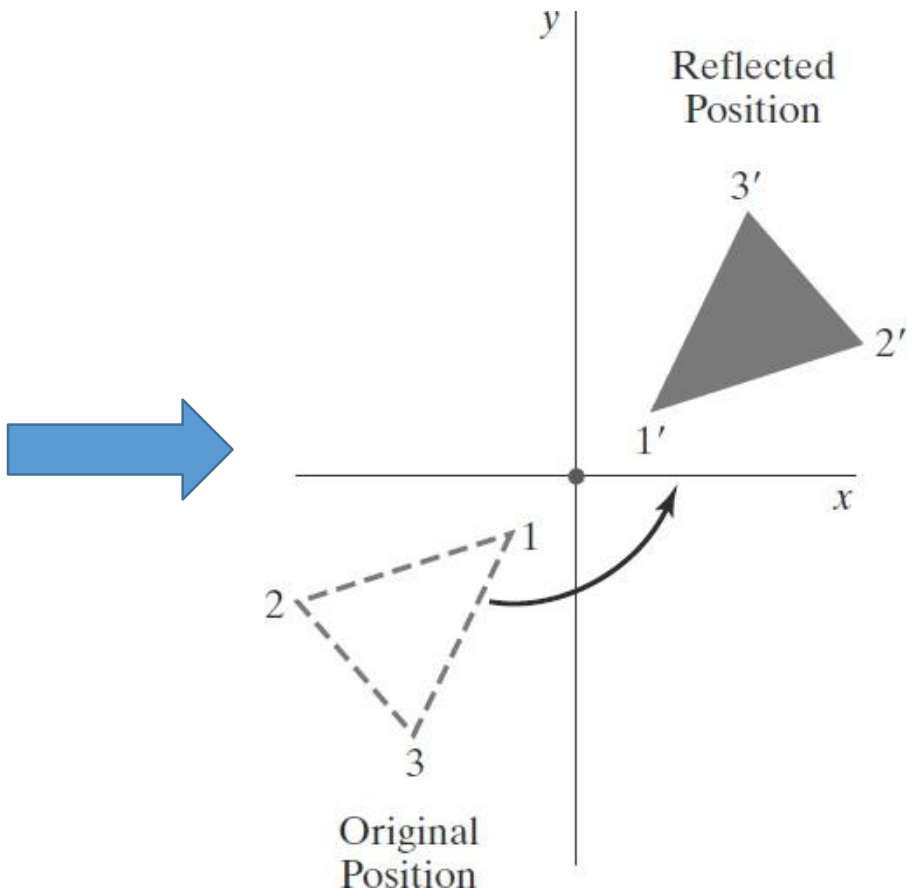
Reflection of an object about Y-axis.

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



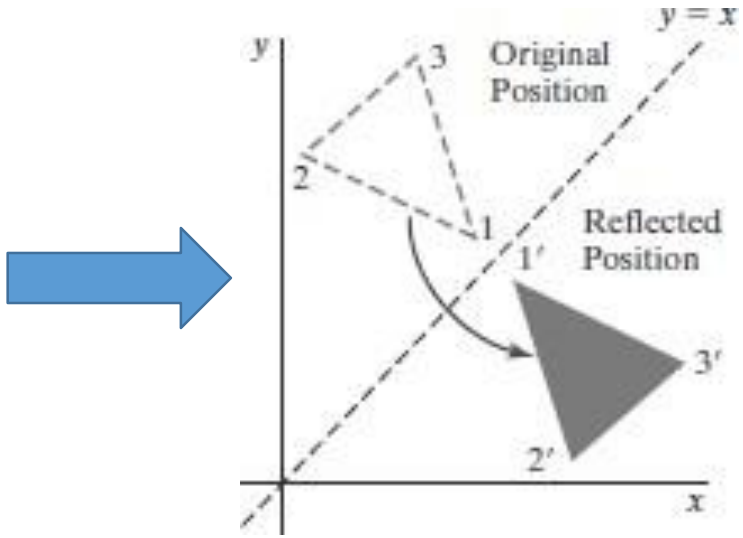
Reflection of an object relative to the coordinate origin.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Reflection of an object with respect to the diagonal line where y = x

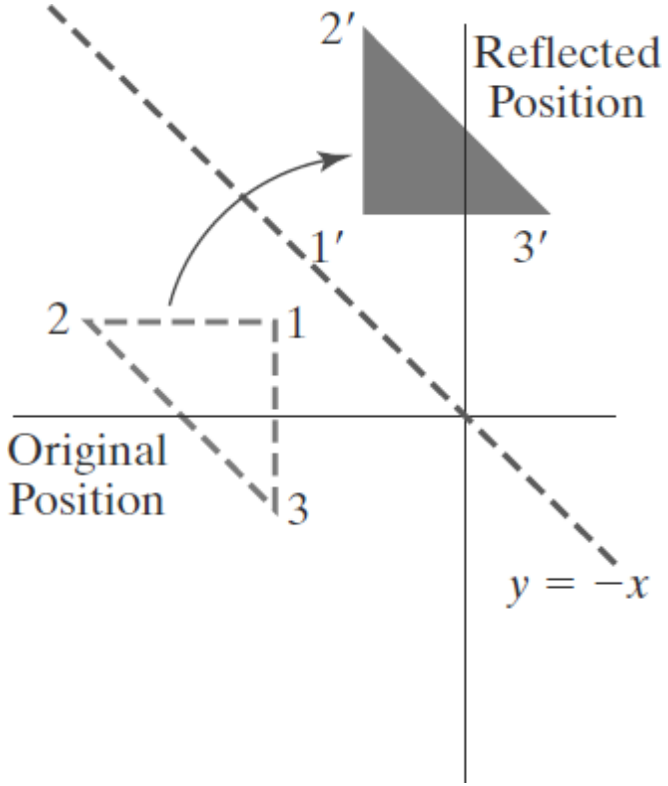
$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



Reflection of an object with respect to the line y = - x

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





Example

Given a triangle with coordinate points a(3,4), b(6,4), c(5,6).. Assume that a reflection on the X-axis is applied.

1. Write down the reflection matrix
2. Use the reflection matrix to calculate the new positions of points a, b, and c

Solution

C= (5,6)

A= (3,4) $\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$ **A’ = (3, - 4)**

B= (6, 4) $\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 1 \end{bmatrix}$ **B’ = (6, - 4)**

C= (5,6) $\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \\ 1 \end{bmatrix}$ **C’ = (5, - 6)**

Example

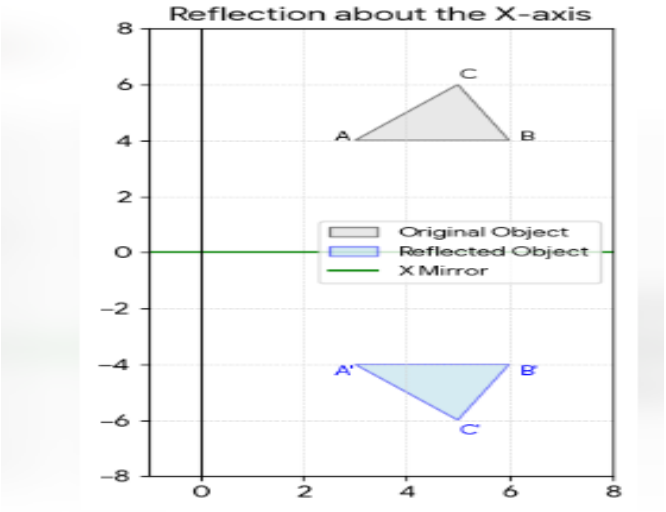
Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection transformation:

Solution

1-about the x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \longrightarrow \text{Reflection matrix}$$

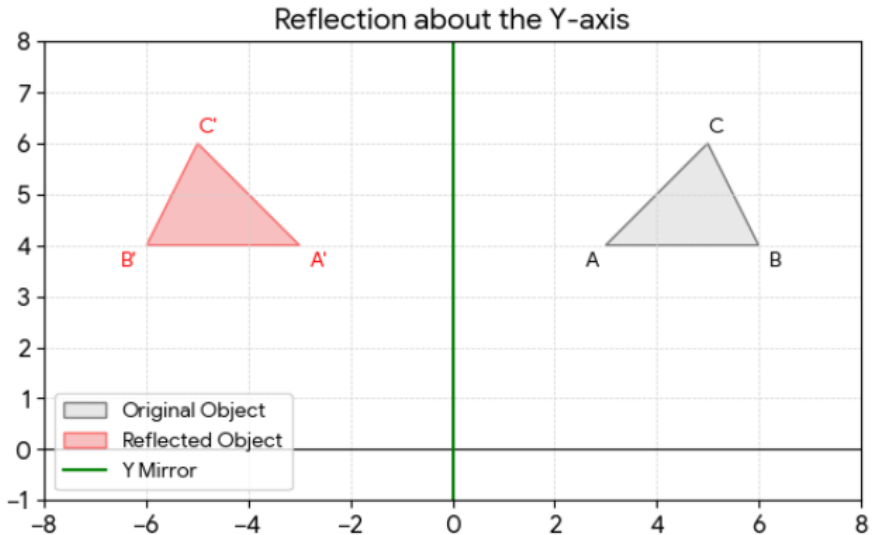
$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 6 & -4 \\ 5 & -6 \end{bmatrix}$$



2- about the y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \text{reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -6 & 4 \\ -5 & 6 \end{bmatrix}$$



Example

Given a triangle with coordinate points A(3, 4), B(6, 4), (C(5, 6). Apply the reflection transformation:

1. about the origin
2. about the diagonal line y=x
3. about the diagonal line y=-x

Solution

1-about the origin

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow \text{reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ -6 & -4 \\ -5 & -6 \end{bmatrix}$$

2-about the diagonal line y=x

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \text{reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 4 & 6 \\ 6 & 5 \end{bmatrix}$$

about the diagonal line y=-x

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \rightarrow \text{reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ -4 & -6 \\ -6 & -5 \end{bmatrix}$$